On Jacob Klein's *Greek Mathematical Thought and the Origin of Algebra*

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Leo Strauss characterized Jacob Klein's *Greek Mathematical Thought and the Origin of Algebra*[1] by using the following words: "The work is much more than a historical work. But even if we take it as a purely historical work, there is not in my opinion, a contemporary work in the history of philosophy or science or in 'the history of ideas' generally speaking which in intrinsic worth comes within hailing distance of it."[2] The purpose of this article is to provide an informed opinion as to what Strauss could have meant by this praise of Klein's book.

Leo Strauss had a confidence, born of his wide-ranging studies, reinforced by a clearly argued need, that political philosophy could pursue its most important questions without any apology to modern political science. He argued that modern political science, or "the scientific study of politics," is unable to substantiate its claims of having bettered its older counterparts. Its failings are, according to Strauss, manifest.[3] One of its most serious failings is that it is guilty of making a "surreptitious recourse to common sense," a recourse disallowed by its scientific hypotheses, which robs it of its scientific pretensions (p. 318).

Strauss argued that we have access to the "political things"—to the primary "data" of political experience yielded by common sense—but that this access is barred to the "new political science." Of even greater significance, Strauss argued that the "naivete of the man from Missouri," that is, the "primary awareness" of human experience, is of a character "that there is no possible human thought which is not in the last analysis dependent on the legitimacy of that naivete and the awareness or the knowledge going with it."[4] And so, however impressive the programmatic promise of the "new political science" might be, Strauss was clear that it could not satisfy our pressing need to address fundamental questions. It could not render an openminded approach to the Ancients illegitimate, and therefore prudence, if nothing else, dictated that we could turn to the Ancients, whose perspective was that of the citizen (WIPP, p. 310). Strauss was not willing to fiddle while burning issues remained unaddressed.

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Strauss was clear that the "new political science" did not by itself stand in the way of returning to the Ancients with an open mind. Rather, what stands in our way is the ubiquitous belief that "the success of modern natural science" makes premodern thought implausible, dependent as it appears to be on cosmological views inconsistent with modern natural science (WIPP, p. 36). To begin with, Strauss could elegantly dismiss this claim by pointing out that classical political philosophy at its "foundation," that is, in its original incarnation at the hands of Socrates, did not depend on "a solution to the cosmological problem." Rather, its theoretical openness was vouchsafed because it was open to "the quest for cosmology," rather than predicated on any particular solution. This elegant simplicity appears to this reader to find sufficient warrant in the manner by which Strauss characterizes what it takes to raise the modern cosmological objection to classical political philosophy. The objection is simple-minded. According to Strauss it "requires neither originality nor intelligence, nor even erudition" (WIPP, p. 36) But Strauss did not leave it at meeting a simple-minded objection with an elegantly simple rejoinder. To this rejoinder he added one of his rare reflections on the character of philosophy which identified it, at its highest, as an openness to the "problem of cosmology." He proceeded to "articulate" this problem as being intellectually open to the dialectic of homogeneity and heterogeneity. (These have always been the primary categories of philosophy, Identity and Difference in Hegel's language, or Sameness and Otherness in Plato's.) The problem of cosmology can be paraphrased as being open to the twin temptations of "absolutizing either knowledge" of a homogeneous, mathematical sort or of knowledge of a heterogenous sort, such as that of the ends of human life, without yielding to either. This philosophical coda points, by my lights, on the one hand to the need for a new look at Jacob Klein's *Greek Mathematical Thought and the Origin of Algebra*, and on the other to the difficult issue of attempting to defend post-Socratic classical political philosophy as it is spelled out by Aristotle.

First Klein: The argument of Klein's book provides profound evidence that modern natural science, at its ontological nerve, is a radical vision of the charm of homogeneity. As well, Klein's book offers an explanation for "the success of modern natural science" which provides independent evidence for Strauss's claim that ancient political philosophy at its foundational level is immune to the challenge posed by modern science. This explanation does justice to the undeniable achievements, both theoretical and practical—in every sense of this distinction—of modern natural science. Here is part of what Klein has to say about the role of modern natural science in our Cave: "Mathematical physics is the most important part of our entire civilization and actual life. This is not only in respect to the technics so inseparable from our modern life, and not only because it determines our own understanding of the world, but also because the principles of mathematical physics are basic to our whole way of thinking and behavior." So that: "We appear to be in the most direct contact with the world around us, but in reality the vast machinery of our society permits us to perceive the world only through generally accepted views."[5] In other words, Klein's book not only reveals the grounds for the success of modern natural science, it also allows one to see that, however ubiquitous and doxastically successful, the objections derived from modern natural science need not, in principle, detain us from the open-minded study of the Ancients at their foundational level. What of Aristotle?[6]

Doxa is neither intelligent nor original nor erudite. But it is powerful, requiring intelligent, original, and profound arguments, such as Strauss's pithy characterization of philosophy paraphrased above, to begin the process "so that our minds may be free."[7] It is not surprising that modern natural science, which, as Klein's book shows, is a radical version of but one option of genuine philosophizing, can close off alternative views which appear to be linked more closely to cosmological solutions than those of Socrates. No wonder there is a widespread belief that attempts to revive ancient political philosophy are at bottom Quixotic because modern *doxa* assert that the quarrel between Ancients and Moderns has been resolved in favor of the Moderns at the level of First Principles, concerning Being and the whole of things. This widespread belief is most effective when Aristotle's version of political philosophy, rather than its Socratic counterpart, is confronted by modern thinking. However much Aristotle's political science may be autonomous on Aristotelian grounds, as shown by Strauss, it appears infected by theoretical premises of a teleological sort.[8]

Jurgen Habermas has expressed the thought in this way:

> . . the ethics and politics of Aristotle are unthinkable without the connection to (his) physics and metaphysics. . . . But today it is no longer easy to render . . . this metaphysical mode of thought plausible.[9]

Strauss was not blind to this difficulty:

> [Ancient political philosophy] in its classic form is connected with a teleological view of the universe. . The teleological view of the universe, of which the teleological view of man forms a part, would seem to have been destroyed by modern natural science.[10]

But notice that Strauss expressed himself modally. Hence a question suggests itself. From what perspective "would" it "seem" that modern natural science destroys the teleological view of the universe? One possibility is that the perspective in question belongs to received opinion of our day, as the observation of Jurgen Habermas seems to confirm. Is it not part of our *nomoi* that modern natural science has resolved the debate at the level of First Philosophy in favor of the Moderns? But is modern natural science authoritative in the full sense of the word at the level of First Philosophy? Does it resolve the debate between Ancients and Moderns? Jacob Klein's work provides an answer to this question which squarely confronts the conceptual integrity of the core of modern natural science in its authoritative mode, namely as mathematical physics. It is an answer which, we will see, provides compelling grounds that modern natural science in its authoritative mode is questionably authoritative, thereby providing evidence that the fundamental questions of First Philosophy remain as accessible in our day as in antiquity, and in addition, it is an answer which affirms that the debate between Ancients and Moderns remains unresolved. With all of that, we need now to turn to seeing in what way Klein's book has a transhistorical-intrinsic, i.e., philosophic, merit because it provides a needed perspective on the struggle between Ancients and Moderns.

The science of nature in its modern incarnation is authoritative with respect to First Principles or metaphysics through the claims of mathematical physics. Adumbrated by Descartes in a fable of the world in his *Discourse on Method* (Part VI), then converted into a commonplace of popular understanding through the course of the nineteenth century, apotheosized in current philosophy of science as the "Reduction Thesis," is the hypothesis that modern natural science, in all of its manifestations, is ontologically dependent on mathematical physics.[11] The "Reduction Thesis" asserts a complex correspondence between science and the world. The world, in ascending order of complexity, is composed of elementary particles (states of energy), higher, more complex, structures such as those observed by chemistry, yet more complex ones such as organisms, and, lastly, man and his institutions. Analogously, the sciences can be rank-ordered in corresponding fashion with mathematical physics at one end and, at the other, the sciences concerned with the human, sociology, psychology, and political science, among others. It is not just the new method of the physical sciences which warrants the scientific character of the modern science of politics. Just as ontologically or in actuality the world is, in the final analysis, "mathematical," so the sciences (if the "Reduction Thesis" is a guide to modern *nomoi*) make contact with the world through mathematical physics. And, as we will see, Klein takes us a long way in understanding a deep-seated conceptual connection between method and ontology in modern consciousness which vouchsafes this dual authority of modern natural science in our Cave.

Accordingly, mathematical physics in its authoritative mode gives us an account of reality. By appealing to the Aristotelian distinction between essence and accident, this point can be fixed with greater accuracy, although, to be sure, the distinction is undermined by mathematical physics. The authoritative status of mathematical physics turns on its ability to give us an account of the essential character of the world, rather than merely describing some of its accidents, even if its account yields proper accidents, even if, in addition, it is operationally successful through the technological successes of modern natural science. If it does not give us an essential account, if it is merely an accidental or an operational account, then it makes no prima facie claims which resolve the debate between Ancients and Moderns in favor of the latter. Can mathematical physics make such an essentialist claim? On the one hand, it must give us an account of what there is and its character in an essentialist mode, providing us with an unambiguous answer to the question as to the "stuff," to use a modern philosophical metaphor, of the world. In its authoritative mode mathematical physics does so. Its answer has been formulated in various ways. Allow this formulation: 'to be' means to be determinable, essentially, in mathematical terms, a formulation which, at the origins of mathematical physics, embraces such diverse forms as Cartesian extension and Newtonian absolute space. On the other hand, the mathematical expressions of mathematical physics, which exhaustively convey the meaning of the physics, indeed all of mathematics whether pure or applied, can be done, from conception to proof procedures, without reference to the world or any standard of "external reality." By any standard of identity ascriptions, identical results can be reached at the same time in any part of the world without reference to the world. But as Sir Arthur Eddington pointed out, mathematical physics seeks to give us an account of the world by "identifying" the mathematical character of its statements with the essential attributes of the world.[12] Is such an identification maintainable? Klein's account of the conceptual structure of modern mathematics suggests that such identification cannot be made as a matter of course and hence it follows that the debate between Ancients and Moderns remains unresolved. Therefore political philosophy in all of its premodern forms can be approached autonomously, and issues of the human good—such as "What is the best life?"—are not barred from the primacy accorded to them by Socrates, Plato, and Aristotle.

The nerve of Klein's inquiries is the claim that the ancient and modern understanding of mathematics, while focusing on the same insight concerning the nature of number, nonetheless differ radically in modes of "conceptualization" (Book, pp. 117–25; cf. Article, pp. 1–5). This word has both a broad and a narrow meaning for Klein. In its broad meaning it includes the concepts which inform a world view, or, to mix ancient and modern similes, "conceptualization" in this sense refers to the horizons defining this or that Cave, city, *nomos*, civilization, or age. Here are included, for example, Klein's reflections on the epochal change and continuity of the concept *episteme* into its modern form, science, through modifications of the concept *scientia*: as with "religion," a word which originally names an affect or predicate of individual human beings, *episteme* or science becomes, under the aegis of modern "conceptualization," a word whose primary reference is more akin to an institution in the modern sense (Book, pp. 118–19). Klein's comparison of modern law with ancient analogy also suggests worlds about "conceptualization" in this broad sense (Article, pp. 28–34). What concerns us here, however, is the narrow sense of "conceptualization." This includes the semantic and ontological implications of the operations of the mind as it deals with concepts, and, as well, reflections on these operations. It is one of the merits of Klein's studies that they never allow the reader to lose sight of the ontological reference of the concepts he examines, and, more importantly, never to lose sight of the hidden or implicit reference of concepts whose customary interpretations bypass this issue. (One result of Klein's reflections, for example, is that they allow one to grasp in what sense the notion of a "concept" has been completely assimilated by modern conceptualization.)

According to Klein, the Greek concept of number has a meaning which, when considered by First Philosophy, yields an ontology of one sort. The modern concept of number, on the other hand, while remaining initially faithful to this meaning, yields on reflection an ontology of a radically different sort.

For the Greeks and the tradition subsequent to them, number, the Greek *arithmos*, refers, always, to a "definite number of definite things." Five or *cinq* or *penta* can refer to either five apples or five people or five dots, but it must refer to a definite number of definite things. Klein quotes Alexander, one of the Aristotelian commentators, "Every number is of something" (Article, p. 23 and n. 24; Book, p. 48). As for counting per se, it refers to things or objects of a different sort, namely monads or units, that is, to objects whose sole feature is unity. Allow an illustration of what this entails: it would be as unthinkable for an ancient mathematician such as Diophantus to assume that an "irrational ratio" such as pi, which is not divisible by one, is a number as it is for us moderns to divide a number by zero. (The neologism, irrational ratio, only means a ratio which yields, in our terminology, an irrational number.) Analogous considerations hold for geometry. A triangle drawn in sand or on a blackboard, which is an "image" of the true object of the geometer's presentation, refers to an individual object, for example, to triangle per se, if the presentation concerns the features of triangle in general. For the Greeks, the objects of counting or of geometry are, if considered by the arithmetic or geometrical arts, in principle, incorporeal. Hence a question arises as to their mode of existence. At least two answers to this question stand out, Plato's and Aristotle's, and, whatever the differences between them, they are agreed on this: to account for what it means to say that there are pure monads or pure triangles must begin from the common ground which has been condescendingly called "naive realism."[13] For Plato, pure monads point to the existence of the Ideas, mind-independent objects of cognition; for Aristotle, monads are to be accounted for on the basis of his answer to the question "What exists?" namely mind-independent particulars, like Socrates, and their predicates, that is, by reference to substances and their accidents.

In order to fix this common ground shared by Plato and Aristotle and thus by the ancient mode of conceptualization, Klein appeals to the language of the Scholastics. According to the Greeks, number refers directly, without mediation, to individual objects, to things, whether apples or monads. It is, in the language of the Schools, a "first intention." Number, thus, is a concept which refers to mind-independent objects. In order to understand Klein's interpretation of the modern concept of number, it is useful to say a few words about the distinction between first and second intentions.[14]

"First intention" is a semantic label for predications such as: 'Socrates is a man,' 'Socrates is an animal,' 'Socrates is pale.' It not only serves as a semantic label for such locutions, it also characterizes their ontological reference. Or, using an appropriate terminology which captures the ontological meaning of "first intention," each of the predications listed above has as an object of reference a first intention; in Aristotelian terms a substance, e.g., Socrates. "Second intention" refers not to things but to concepts. "Second intention" is a semantic/ontological label for predications such as: 'man is a species,' 'animal is a genus,' 'pale is said of individuals.' Or, each of the latter has as an object of reference a second intention; in modern terminology, a concept. One way of characterizing the difference between first and second intentions is to say that 'man,' 'animal,' 'pale' are united in one thing, Socrates; while species, genus, predicate exist in and are separate in or through thought. Accordingly, the object of a first intention, at least as illustrated in these examples, is easily imaginable; whereas in the case of the objects of the second intentions above, they are literally unimaginable, only instantiable. In our ordinary way of speaking, the difference between first and second intentions is a difference in abstractness. But, as we will see, Klein's studies suggest that not only is abstractness misapplied in this case, but that, as well, the modern concept of number stands between us and an appreciation of why this is so. Finally, we note: the Greek concept of number, *arithmos* as instantiated in, say, *penta*, is a first intention, i.e., it refers to mind-independent entities, whether it is apples or monads.

The modern concept of number results from what Klein calls a "symbol generating abstraction" (Book, p. 202). What this entails is the identification, with respect to number, of first and second intentions. From the point of view of "naive realism" or ancient ontology this is, strictly speaking, an oxymoronic endeavor. In order to make sense of the notion of a symbol-generating abstraction, we need to go through, in outline, Klein's account of the modern concept of number. But at the outset, let us spell out what Klein's studies show: (1) Symbolic mathematics, as in post-Cartesian algebra, is not merely a more general or more abstract form of mathematical presentation. It involves a wholly new understanding of abstraction which (2) implies a wholly new understanding of what it means for the mind to have access to general concepts, i.e., second intentions, as well as (3) implying a wholly new understanding of the nature and mode of existence of general concepts, and (4) Klein's inquiries do genuine justice to the concept of variability or generality which is so important to symbolic mathematics and which is at the heart of the most important achievements of modern natural science, achievements which are fully recognized by Klein.

Klein's account of the modern concept of number is based on readings of the mathematical and philosophical works of such diverse sixteenth- and seventeenth-century figures as Vieta, Stevin, Descartes, and Wallis. We will first consider Klein's interpretation of Vieta's *Isagoge* and then turn to his account of Descartes. This will first lay bare the semantic and ontological implications of the new mathematics; on turning to Descartes we will examine the new understanding of the working of the mind implied by Vieta's revolutionary interpretation of the concept of number.

According to Klein, Vieta's conception of number, while it is the starting point of the modern concept of number, nonetheless still begins with the traditional understanding of the *arithmos* concept. In short, we are confronting a different interpretation of the same phenomena; ancient and modern modes of conceptualization share a common ground within human experience which does not account for their ultimate divergence, thereby providing independent confirmation, from an unexpected direction, of the judiciousness of Strauss's defense of the unavoidability of the "naivete of the Man from Missouri." In order to display where Vieta departs from the ancient mode of conceptualization, Klein focuses on the use of letter signs. Klein's patient exegesis dispels the hazy notion that a letter sign is a mere notational convenience (a symbol in the ordinary sense of the word in our day) whose function it is to allow for a greater generality of reference. Rather, Klein argues, symbol, as he interprets the character of "symbol generating abstraction," entails a wholly new mode of ontology and conceptualization.

Every number refers to a definite multitude of things, not only for ancient mathematicians but also for Vieta. The letter sign, say, 'a,' refers to the general character of being a number, however, i.e., not to a thing or a multitude of things, but, instead, to a concept taken in a certain manner, that is, its indeterminate content. In the language of the Schools, the letter sign designates a second intention; it refers to a concept. But note what is of critical importance according to Klein; it does not refer to the concept number per se but rather to its 'what it is,' i.e., to "the general character of being a number."[15] The letter sign, 'a,' in other words, refers to a "conceptual content," i.e., mere multiplicity, which, as a matter of course, is identified with the concept (Book, p. 174). This matter-of-course, i.e., implicit, identification is the first step in the process of "symbol generating abstraction." According to Klein, this step, which is entailed by Vieta's procedures—not, we should stress, merely entailed by Vieta's reflections on his procedures—makes possible modern symbolic mathematics. In other words, at the outset, at the hands of its "onlie begetter," Vieta, the modern concept of number suggests a radical contrast with ancient modes of conceptualization.

For Plato and Aristotle *logos*, discursive speech, is man's communal access to the *definiens* of a concept, i.e., its "content."[16] Not so for modern conceptualization. The letter sign refers, gives us access to, "the general character of being a number," mere multiplicity. (Although it was left to Descartes, in

Klein's view, to work out the implications of this mode of conceptualization.) In addition, the letter sign indirectly, through rules, operational usages, and syntactical distinctions of an algebraic sort, also refers to things, for example, five units. This leads directly to the decisive and culminating step of "symbol generating abstraction" as it emerges out of Vieta's procedures. It occurs, according to Klein, when the letter sign is treated as independent, that is, when the letter sign, because of its indirect reference to, say, things or units, is accorded the status of a first intention, but—and this is critical—all the while remaining identified with the general character of a number, i.e., a second intention. Klein sums up this momentous achievement: a potential object of cognition, the content of the concept of number, is made into an actual object of cognition, the object of a first intention.[17] The signal character of this achievement needs to be spelled out. From now on, number is both independent of human cognition, i.e., objective, *and* without reference to the world or any other mind-independent entity, which, from the point of view of the tradition—if not common sense—is paradoxical.

All of this means, according to Klein, that "the one immense difficulty within ancient ontology, namely to determine the relation between the 'being' of the object itself and the 'being' of the object in thought is . . . accorded a 'matter-of-course' solution . . . whose significance . . . (is) . . . simply-by-passed" (Article, p. 192). Allow a few more details. The mode of existence of the letter sign (in its operational context) is symbolic. Let us try to grasp Klein's suggestion about what symbolic means by contrasting it with the Platonic and Aristotelian accounts of mathematical objects. For Plato the correlate of all thought which claims to be knowledge is the mind-independent form, or idea, or genus, or, in the case of number, monad; none of these are the ontological correlates of the symbolic, modern, grasp of mathematics. For Aristotle the object of the arithmetical art results from abstraction, but abstraction understood in a precisely defined manner which, when examined, shows that the mode of existence of the referent of the letter sign of modern mathematics is not abstract in this Aristotelian sense but is, rather, symbolic. Thus, following Klein's interpretation, both symbol and its referent are not only sui generis, arising out of the new understanding of number implied by the algebraic art of Vieta, they are, as well, logical correlates of one another, symmetrically and transitively mutually implicatory of one another. That is, symbol—in "symbol generating abstraction"—is not a place marker which refers, as in the ordinary sense of symbol of our day. Rather it is the logical, conceptual, and thus quasi-ontological correlate of its referent, namely the "conceptual content" of the concept of number, i.e., mere multiplicity.

But to return for a moment to Aristotle: the issue addressed by his discussion of abstraction (*aphairesis*) is to account for the purity and mode of existence of the referent of *arithmos*. *Aphairẹsis* serves as an answer to both concerns. The purity of the monad results from the leaving out of consideration all other sensible qualities of things, i.e., all accidents of substances, and retaining only those accidents or predicates which fall under the category of quantity. "Leaving out of consideration" and "retaining" are what Aristotle calls abstraction (*Metaphysics* K3, 106a28ff; Book, p. 105). It is no more a psychological account of the genesis of number than the *Categories* is a psychological account of the genesis of the structure of logos. *Aphairesis* is an ontological-semantic doctrine which, in a manner analogous to the *Categories*, spells out the implications of a concept in a manner that leaves open to inspection the logically correlated metaphysics which supports it. (The possibility of doing a semantics which is metaphysically neutral is a consequence, if Klein is our guide in these matters, of modern conceptualization whose mode of thinking is defined by symbolic mathematics.) Abstraction, as Aristotle understands it, is possible because there are substances, e.g., Socrates, and their accidents, some of which are in the category of quantity. Although quantity is pure for the mathematical arts as Aristotle interprets them, it is no less connected to the world, in Aristotle's account, than, say, Turner's reflections on color are connected to the things of this world. None of this holds for the symbol of modern mathematics. As we have seen, it does not refer to the world, but rather, initially, to the content of a concept, its *definiens*.

In short, the modern concept of number, defined as it is by symbolic procedures, is not merely a continuation of the ancient concept—as is supposed by the modern self-interpretation of mathematics—only carried on at a higher level of abstractness or generality.[18] On the contrary, while bound to the ancient concept, the modern version is, paradoxically, less general. Abstraction in the non-Aristotelian sense, the usual label for symbolic modes of thought, can be grasped in at least two ways. First, it presents itself as a term of distinction as in the pair abstract/concrete. Whereas the concrete stands before us or can be presented through or by an image, the abstract cannot.[19] Alternatively abstract in the modern interpretation can also be illustrated by an ascending order of generality: Socrates, man, animal, species, genus. The scope of the denotation, or the extension, increases as abstractness increases, and, once again, the more general is also the less imaginable. But this is precisely what symbolic abstraction is not. The mathematical symbol 'a' in context has no greater extension than the ancient number, say, *penta*. Rather, the symbol is a "way"—or, in the modern interpretation of method which Descartes inaugurates, a step in a "method"—of grasping the general through a particular.[20] It is a way, if you will, of imagining the unimaginable, namely the content of a second intention, which is, at the same time, through procedural rules, taken up as a first intention, i.e., something which represents a concrete 'this,' or *tode ti*, in the Aristotelian terminology. And, as Klein notes, one consequence of this reinterpretation of the concept of *arithmos* is that the "ontological science of the ancients is replaced by a symbolic procedure whose ontological presuppositions are left unclarified" (Book, p. 184).

The most important consequences of this lack of clarity are open to inspection in Descartes' account of number. According to Descartes, as Klein reads him, the human intellect has a capacity to conceive or grasp, not merely represent, the content of general concepts such as mere multiplicity or the extendedness of extension. This arises from the mind's ability to deal with and reflect on its own capacity to know. It can comprehend fiveness or even multiplicity in general as separate from five counted points or other objects either corporeal or pure. But this power to abstract is not Aristotelian abstraction. Rather this claim, Klein notes, entails a "new mode of 'abstraction' and a new possibility of understanding" (Book, pp. 200 and 199–202). This "new mode" of abstraction may properly be called second-order or meta-abstraction. It is an abstractive capacity which does not deal with things and their properties but rather with concepts, or abstract entities in the ordinary usage of the term. Descartes' suggestion that the mind has such a power answers to the requirements of Vieta's supposition that the letter sign of algebraic notation can refer meaningfully to the "conceptual content" of number. The "new possibility of understanding" required is, if Descartes is correct, none other than a faculty of intellectual "intuition."[21] But this faculty of intellectual intuition must not be understood in terms of the Kantian faculty of intellectual intuition. The Cartesian version, implied by Descartes' account of the mind's capacity to reflect on its knowing, unlike its Kantian counterpart, is not informed by an extra-mental object. (Of course, since for Kant the human intellect cannot intuit extra-mental objects in the absence of sensation, there is no human faculty of intellectual intuition. It is, for Kant, a faculty *per impossibile* which illustrates a limitation on human knowing.)[22]

Moreover, this power of intuition, in Klein's words, has "no relation at all to the world . . . and the things in the world" (Book, p. 202). In other words, it is not to be characterized so much as either incorporeal or dealing with the incorporeal but, rather, as unrelated to both the corporeal and the incorporeal, and so perhaps is an intermediate between the "mind and body," the fulcrum of traditional interpretations of Descartes.[23] In the simplest terms, the objects of mathematical thought are given to the mind by its own activity, or, mathematics is metaphysically neutral. Nonetheless, this unrelatedness of mathematics and world does not mean that mathematical thought is—like Aristotle's Prime Mover—merely dealing with itself alone. It requires, according to Descartes, the aid of the imagination. The mind must "make use of the imagination" by representing "indeterminate manyness" through symbolic means (Book, p. 201). A shift in ontology, the passage from the determinateness of *arithmos* and its reference to the world, even if it is the world of the Forms, to a symbolic mode of reference becomes absorbed by what appears to be a mere notational convenience, its means of representation, i.e., letter signs, coordinate axes, superscripts, etc., thus preparing the way for an understanding of method as independent of metaphysics, or of "ontological commitment," in the language of the schools of our day. The conceptual shift from *methodos* (the ancient "way," particular to, appropriate to, and shaped in each case by its heterogeneous objects) to the modern concept of a "universal method" (universally applicable to homogeneous objects) is thus laid down. The way is prepared for a science of politics whose methodology is scientific and whose influence was one of the main polemical objects of Leo Strauss's work.

The interpretation of Vieta's symbolic art by Descartes as a process of meta-abstraction by the intellect, and of representation of the abstracted for and by the imagination is, then, what Klein calls "symbol generating abstraction" as a fully developed mode of conceptualization (Book, pp. 202, 208; cp. pp. 175, 192). Consider two results of this intellectual revolution.

1. In order to account for the mind's ability to grasp concepts unrelated to the world, Descartes introduces a separate mode of knowing which knows the extendedness of extension or the mere multiplicity of number without reference to extra-mental objects universal or particular. This not only allows, it logically implies, a metaphysically neutral understanding of mathematics. A mathematician in Moscow, Idaho, and one in Moscow, Russia, are dealing with the same objects although no reference to the world, genetic or ontological, needs to be imputed.[24]

2. "Symbol generating abstraction" yields an amazingly rich and varied "realm" (to use Leibnitz' sly terminology) of divisions and subdivisions of one and the same discipline, mathematics. For confirmation, one need only glance at the course offerings of a major university calendar under the heading "Mathematics." Yet the source of this "realm" is at once unrelated to the world and deals with the "essence" of the world through mathematical physics in its essentialist mode. For the Descartes of the early "scientific" works, inclusive of all of the foundational arguments examined by Klein, this is possible because the imagination is Janus-like. It is the medium for the symbol and also a bridge to the world, since the world and the imagination share the same "nature," i.e., corporeality or, what comes to the same thing, the "real nature" of corporeality, extension.[25]

In a lecture entitled, "Progress or Return?" Strauss spoke of "the amazing vitality of the West" whose intellectual content, at once its core and life, is animated and propelled by unresolved tensions.[26] Nowhere is this vitality more in evidence than in the ways in which the Tradition of the West extended, at times accretionally, at other times through bold leaps, ways of understanding laid down by the ancient Greeks. From the nominalist solutions of the problems of limits and aggregates, to the reinterpretation of the *arithmos* concept in the sixteenth and seventeenth centuries, to name but two examples, a certain restlessness has animated the Tradition's reception of its own presuppositions.[27] But the cunning of reason, or chance, has had a role to play here. Klein points out that Vieta for one, as well as Fermat, simplified their achievements. They understood the "complex conceptual process" of symbol-generating abstraction as merely a higher order of "generalization," thereby setting the stage for what has come to be habitual for modern consciousness, the passing over of the theoretical and exceptional, so that, in Klein's phrase, it is simply "by-passed" (Book, p. 92). (All this is an almost uncanny inversion of Heidegger's insistence that the passing over of the proximal and everyday must be overcome to appropriate Being in our day.) But this blindness to its own achievements, from which the modern science of nature suffers, is a condition of its success. Only if the symbol is understood in this way—merely as a higher level of generality—can its relation to the world be taken for granted and its dependence on intuition be "by-passed." Only if symbol is understood as abstract in modern *doxa*'s meaning of the word would it have been possible to arrive at the bold new structure of modern mathematical physics on the foundations of the old.

It is important to grasp the conditions of the success of the modern concept of number. One of these is that modern mathematics is, to repeat, metaphysically neutral.[28] This means, first of all, that modern mathematics does not entail, of itself, or presuppose of itself, metaphysical theses concerning what exists or what is the meaning of Being. For a contrast, one need only follow Klein's patient exegesis of Diophantus' *Arithmetic*; there, object, mode of presentation, scope of proof, and rigor of procedure are intermingled with metaphysics (Book, pp. 126–49). As one commentator has pointed out, Klein shows that "Aristotle's theory . . . of mathematical concepts . . was assimilated . . . by Diophantus and Pappus."[29] Secondly, and more conclusively, the proofs and content of modern mathematical arguments need not be considered in conjunction with the metaphysical orientation of the mathematician presenting the argument, and so, mutatis mutandis, whereas the premodern world could distinguish between Platonic and, say, Epicurean physics, no analogous distinction is viable in the modern world. There is yet a third way in which modern symbolic mathematics is metaphysically neutral, and this in the strongest sense. It is neutral because it is all consistent with all metaphysical doctrines, nominalist or realist, relativist or objectivist. Whatever the metaphysics, to date, there is an interpretation of modern mathematics which leaves it unscarred.[30] This is not the case for the ancient conception. For example, Euclid's division of the theory of proportions into one for multitudes and another for magnitudes is rooted in the nature of things, in an "ontological commitment" to the difference between the two. Only after the metaphysical neutrality of the modern conception is taken for granted and bypassed, is it possible to do away with Euclid's division as a matter of notational convenience.

None of this, of course, holds true for mathematical physics in its authoritative mode, as arbiter of what there is, that is, in the version it must assume to serve as a ground for the prima facie acceptance of the victory of Moderns over Ancients at the level of First Principles. Mathematical physics does make—in this mode—metaphysical claims. It is not metaphysically neutral. Elementary particles are, for example, if mathematical physics is arbiter of what there is.

But are they? Take, to begin with, the most influential version of ontology extant for those who accept the Reduction Thesis, that is, Willard Van Orman Quine's famous dictum that "to be means to be the value of a bound variable."[31] Drawn as the dictum is in order to make metaphysics safe for physics, does it entail the existence of, say, elementary particles? Assuredly not; after all, Quinean ontology can only inform us about the semantic conditions of ontological statements. All we know, accordingly, is that if we claim that particles are—that is, are *in re* and not merely operationally defined—then our claim will fit this semantic model. Conversely, sets, aggregates, mathematical infinities also qualify as "existents" in this semantic sense, but they cannot give us any knowledge of the world, since we need not impute to them any extramental reference when we deal with them as pure objects of mathematics. In other words, as long as, in Cartesian terms, the identification of the real nature of body as extendedness with the objects of mathematical thought remains unproven and is merely, in effect, asserted, Sir Arthur Eddington's hope that mathematical physics gives us an essentialist account of the world will remain just that.

All of the above means that Klein's book is a key to understanding modernity's most profound *doxa* about the nature of Being, of bringing to light the very character of these modern *doxa* in a manner which discloses not only their historical genesis but lays open to inspection why they are not only *doxa* but also *nomoi*. Thus the book is a key to renewing that most daunting of human tasks, liberating us from the confines of our Cave. For example, it is entirely possible, in a stronger than logical sense of the word, that, however daunting the prospect might be, we can leave aside the anti-teleological bent of our Cave and entertain the contrary bent of political philosophy in its classic form, and so pursue without fear of metaphysical bad faith the entire complexity of Aristotle's reflections on the human things.

As for the "intrinsic" importance of Klein's work, allow the following suggestion based upon what has been examined in this article by way of an answer to what Strauss could have been pointing to. There is a temptation to distance oneself from modern natural science by characterizing it as a kind of Poetry, i.e., an enterprise which identifies making and knowing in a way that obscures the objectivity of truth. By toying with the possibility that modern natural science is a form of Poetry, a modern instantiation of one side of the quarrel between Philosophy and Poetry, we also toy with the possible supremacy of an unmistakably Nietzschean if not Heideggerian metaphysical orientation. Jacob Klein's work allows for a shift in perspective which rescues us from both the pan of mathematical physics and the fire of "historicism" in its most radical version, the will to believe that Poetry reigns supreme. Modern physics cannot only be viewed as Nietzschean, or, even, Leibnitzian—committed to a "harmony" between mind and world—it can also be characterized as unwittingly Parmenidean, committed to the identification of Thought (symbolic mathematics) and Being (elementary particles). In these terms, Klein's work provides us with a compelling argument that Parmenides' dictum:

to gar auto noein te kai estin
(For Thinking and Being are the same.)

is, as it always was, as much of a statement of a fundamental philosophical problem as the solution to one. The success of modern physics, practical and theoretical, underlines the need for continual reflection on its so-far-unexplained equation between Being and Thought. Klein's work reminds us that the evidence for the fundamental questions remains accessible even in our day, even as in antiquity. Truly, at the level of First Philosophy the struggle between Ancients and Moderns remains unsettled.

NOTES

1. Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (Cambridge, 1968), henceforth Book; and "The World of Physics and the 'Natural' World," henceforth Article, in *The Lectures and Essays of Jacob Klein*, ed. Williamson and Zuckerman (Annapolis, 1985), pp.1–34; henceforth Essays.

2. Leo Strauss, "An Unspoken Prologue to a Public Lecture at St. John's," *Interpretation* 7(1980) 3. To the best of my knowledge Strauss never explained what he meant either by suggesting that Klein's book has a transhistorical significance or what he meant by its intrinsic worth.

3. Leo Strauss, An Epilogue to *Essays on The Scientific Study of Politics*, ed. Herbert J. Storing (New York, 1962); henceforth ESSP. Here is a partial list of its failings: it does not provide criteria of relevance worthy of its pretensions to objectivity (p.319); it is characterized by a "formalism unrivalled in any scholasticism of the past"(p.319) which it customarily evades through sheer parochialism (p.320); it seeks a precision in its language heterogenous with the phenomena it seeks to elucidate (p.321); it invariably succumbs to dogmatism with respect to theology (p.322); it relies on suspect theoretical underpinnings such as the fact-value distinction with such a lack of suspicion that it is thereby once again almost unavoidably parochial in character (p.324).

4. ESSP, p. 316. In a book review, Strauss raised the possibility that the "questions raised by . . Aristotelian physics retain their full significance regardless of any progress that modern science has achieved" (*What Is Political Philosophy* [New York, 1959], p. 286; henceforth WIPP).

5. Jacob Klein, "Modern Rationalism," in Essays, pp.53–64, see p.57 for the former, and p.64 for the latter. Cf. also " . even a critical attitude towards mathematical physics does not free us from its dominion" (Article, p. 3).

6. What of Plato? Allow an appeal to authority (*ad vericundiam*) which implies that on the issue of steering clear of cosmological dogmatism Plato remained a "Socratic," as Strauss defines the term in his account of philosophy paraphrased above. According to Klein, the *Philebus* intimates that "*nous* is above all a human possession, and that Socrates is the embodiment of *nous*." See, "About Plato's *Philebus*," Essays, p.342.

7. Leo Strauss, *The Political Philosophy of Hobbes: Its Basis and Its Genesis* (Chicago, 1961), p.xx.

8. On the autonomy of Aristotle's political philosophy see ESSP, p. 309.

9. Quoted in William Galston, *Justice and the Human Good* (Chicago, 1980), p. 11.

10. Leo Strauss, *Natural Right and History* (Chicago, 1965), pp. 7–8.

11. Ernest Nagel, *The Structure of Science* (New York, 1961), pp. 336–80.

12. Sir Arthur Eddington, *The Mathematical Theory of Relativity* (Cambridge, 1924), p. 222.

Stephan Korner, who, on the whole, represents the dominant opinion on this issue, does not agree with Eddington. He argues that the ascription of an identity between mathematics and the world is merely hypothetical or " . as if . . ," which cannot be taken to mean "discovered, conjectured, or postulated" (Stephan Korner, *The Philosophy of Mathematics* [New York, 1960], pp. 177–79). This, of course, does away with the authoritative character of mathematical physics, and so runs counter to modern *doxa*. To be sure, all of this is consistent with Strauss's and Klein's insistence that *doxa* or the "surface of things" is not only at the heart of the matter but hardest to access. The lack of agreement between Eddington and Korner illustrates a fundamental incoherence in modern thought. Klein's work provides a plausible argument that this incoherence is embedded in the founding stages of modernity in the sixteenth and seventeenth centuries. As a result, mathematical physics is, at once, a paradigm for ontology, and mathematics per se is abstract, unrelated to the world and thus only accidentally descriptive, via mathematical physics, of reality. (For a contemporary version of the latter, see Korner, above; for the former, see Willard Van Orman Quine, "On What There Is," in *From a Logical Point of View* [New York, 1961], pp. 1–19; henceforth Quine.)

13. R.G. Collingwood, *An Essay on Metaphysics* (Oxford, 1940), p. 34.

14. See Joseph Owens, *An Elementary Christian Metaphysics* (Milwaukee, 1963), pp. 237–41. For Klein's account of why he appeals to the language of the Scholastics, see Article, pp. 7–8.

15. Book, p. 174, cp. p. 201: the starting point of Cartesian abstraction is the intellect's grasp of the content of a concept, e.g. mere multitude; see also p. 192.

16. Cp. Article, p. 17: ". . . mathematical objects in the Greek sense . . are accessible only to the discursive intellect. "

17. For a contrast with the ancient mode of conceptualization, see Article, pp. 132–33; also p. 123: the "heart of [the] symbolic procedure" is that it identifies object represented with means of representation and it replaces real determinateness of object with possibility of determination. Would it be too fanciful to suggest that the Heideggerian inversion of the actual/potential distinction, with the latter receiving at his hands a higher dignity than the former, finds a root in this symbolic procedure, and with that, all that is entailed for understanding Heidegger as a Modern rather than accepting his self-understanding of himself as true claimant to representing the end of philosophy, a claim which obviates the distinction between Ancients and Moderns?

18. It is noteworthy that the founders of modern symbolic mathematics did interpret their achievements in this way. See Article, p.20: "What Fermat and Descartes call 'generalization' is in reality a complex conceptual process ascending from *intentio prima* to *intentio secunda* while, *at the same time*, identifying these." It is no less worthy of note that this one-sided self-interpretation has become a commonplace of modern *doxa*.

19. One way of indicating the distance between abstract in the modern and Aristotelian sense is to note that the pair abstract/concrete is not isomorphic with the Aristotelian form/matter distinction. For Aristotle the abstracted is also a combination of matter and form, i.e., to be sure, of intelligible matter. See Book, p. 295 n. 314; also cp. Joseph Owens, *The Doctrine of Being in the Aristotelian Metaphysics* (Toronto, 1963), pp. 342–43.

20. The inseparability of method and ontology in the context of linking universal and particular through the symbolic procedures inaugurated by Descares should be kept in mind when reading the description of the "universal" Method in the *Discourse on Method.* Contemporary readings of the Method as prescriptive for those of "good sense," and thus as universally applicable for every conceivable problem, and hence readings which would make it at the best Quixotic, turn a blind eye to Descartes' apologetic or exoteric intention in the *Discourse* (see VI). The subtext of the Method, i.e., its esoteric intention, is to be found in its ontological content, as the point above suggests, and thereby its universality is descriptive, rather than prescriptive, with respect to Descartes' metaphysical claim that only mathematical physics gives us knowledge of the world. See *Discourse on Method*, Part II.

21. Book, p.200. In addition, Klein presents a reading of Stevin which suggests the same result. Stevin's number concept which, in effect, denies the ancient view that 'One' is not a number (a major sticking point for moderns, which seems at once inexplicable and perverse) also requires a faculty of intellectual intuition. Stevin's key premise, that the "material" of a multitude of units is "'number'" implies, as with Vieta, that conceptual content and concept have been identified, cp.

pp. 191–92. Stevin's role in modern conceptualization cannot be overstated. He originates the number line which (a) provides modernity with an unshakeable visual—eidetic in the modern sense—metaphor for the homogeneity of magnitude and multitude, (b) thereby obviating ancient objections to treating "irrational ratios" as numbers—rational and irrational numbers are both "schnitts," to use Dedekind's felicitous formulation—and (c) assimilates both One and Zero as numbers homogenous with all other numbers. Stevin's work does not add to the ontological apparatus of "symbol generating abstraction," however. For a vivid account of these matters, see Klein's "On a Sixteenth Century Algebraist," Essays, pp. 35–42.

22. See Moltke Gram, "Intellectual Intuition: The Continuity Thesis," *Journal of the History of Ideas*, 35, no.1 (April–June 1981), 287–304.

23. See Richard Kennington, "The 'Teaching of Nature' in Descartes' Soul Doctrine," *The Review of Metaphysics*, 26, No. 1 (1972), who makes a persuasive case that the dualism of mind-body cannot be divorced from Descartes' exoteric or apologetic intention.

24. To the best of my knowledge, the term "metaphysical neutrality" was first used in a published context by Leo Strauss: "Prior the victory of the new physics . . . to speak colloquially, there was no metaphysically neutral physics. The victory of the new physics led to the emergence of physics which seemed as metaphysically neutral as, say, mathematics, medicine, or the art of medicine" (ESSP, p.309). Klein provides independent, consistent confirmation of this claim unavailable elsewhere. This article does not mean to suggest that the metaphysical neutrality of mathematics and of mathematical physics, in its nonessentialist mode, is part of the modern world's self-understanding. On the contrary, as Klein suggests, the belief that mathematical physics yields "the inner constitution of nature as Galileo and Boyle" thought is at the heart of our Cave (Essays, p.231). If this were not the case, there would be no need, in my eyes, for making Klein's argument accessible to a wider audience; see below note 28.

25. (1) For the later Descartes, there appear to be a host of concepts—innate ideas, God, continuous creation, clear and distinct ideas, the pineal gland, to name a few—which can be used in interpretations to imply that the doctrine of the corporeality of the imagination of the *Regulae* as the bridge between mind and body is compromised. For contemporary defenses of each of the two Descartes as fundamental, see Stephen Gaugkroger, "Descartes' Project For A Mathematical Physics," who argues, in effect, that the early Descartes is fundamental, and also Martial Gueroult, "The Metaphysics and Physics of Force in Descartes," for a defense of the later Descartes as fundamental; both are in *Descartes: Philosophy, Mathematics, and Physics*, ed. Stephen Gaukroger (Totowa, New Jersey, 1980), pp.97–140 and pp.196–229 respectively. (2) With that, allow a perspective from Straussian premises. According to Strauss, an important *doxa* of our Cave is a separation between philosophy and science which is a "consequence of the revolution which occurred in the seventeenth century" (ESSP, p. 309). Of the two, science, in particular "physics (and mathematics)", is the most successful. If one assumes that the intention of the late Descartes' metaphysical arguments was in part, at least, apologetic, exoteric, meant to protect the new science of nature from hostile forces of an anti-Galilean character, one can surely applaud Descartes' success. (3) No doubt Cartesian "extension" is the bridge between early and late Descartes on this question. The foundation of that bridge on the early side is made clear by Klein: " Descartes' concept of *extensio* identifies the extendedness of extension with extension itself" (Article, p. 21). This identification is made possible, credible, and successful through Vieta's and Descartes' identification of the content of mathematical second intentions, the *definiens*, with the concept itself. As for the foundations of the bridge in the later Descartes, I wait impatiently for a patient reader of a cast of mind like Kennington's to unravel the terminology of the Fourth and Fifth Meditations in order to see if there, perhaps, may be found an ontological doctrine coherent with the early argument.

26. Leo Strauss, "Progress or Return?" in *An Introduction to Political Philosophy: Ten Essays by Leo Strauss*, edited with an introduction by Hilail Gildin (Detroit, MI: Wayne State University Press, 1989), pp.249ff.

27. Curtis Wilson, *William Heytesbury: Medieval Logic and The Rise of Mathematical Physics* (Madison, 1960), p. 148.

28. For more on metaphysical neutrality, see Richard Kennington, "Strauss's Natural Right and History," *Review of Metaphysics*, 42, no. 2 (Sept. 1981), 57–86, esp. 82–85.

29. Hiram Caton, *The Origins of Subjectivity* (New Haven, 1973), p. 168.

30. In light of this, one can better understand why contemporary "Mathematical Platonism" (i.e., the assertion that mathematical entities are mind-independent), although ubiquitous, carries so little permanent impact in today's philosophy of science. It is not merely because, at worst, it is only a pious wish to found mathematics on solid objectivist ground, or because, at best, it is only a restatement of the problem, but, rather, because modern symbolic mathematics is metaphysically neutral, i.e., logically independent of such metaphysical theses. On "Platonism's" ubiquity, consider the following: " it is the dominant attitude . of modern mathematics " (*The Encyclopedia of Philosophy* [New York, 1967], vols. 5–6, p. 201). On its lack of solid argument, or conceptual utopianism, consider also: "Mathematical objects are treated as if their existence is independent of cognitive operations, which is *perhaps evident* . " (ibid., emphasis added). For a discussion of "Mathematical Platonism" see Kurt Godel, "Russell's Mathematical Logic," in *Philosophy of Mathematics: Selected Readings*, ed. Paul Benaceraff and Hilary Putnam (Englewood Cliffs, NJ, 1964), pp. 211–32, esp. pp. 212–13, cp. p. 220, and also Paul Bernays, "On Platonism in Mathematics," ibid., pp. 274–86. For a glimpse of the conceptual distance between contemporary Platonism and the ancient version, not to mention the "Platonism" of Plato's dialogues, see Gottfried Martin, *Leibnitz: Logic and Metaphysics* (New York, 1960), p. 172. Given this distance, contemporary "Platonists" ought rightly to bear the sobriquet of "naive realism"; see note 13 above.

31. Quine, pp. 12 and 15. This Quinian bon mot is modernity's way of acknowledging, albeit unconsciously, reformulating, and thus agreeing with Aristotle's contention that to be means to be a *tode ti*.